

# New-physics signals of a model with a vector-singlet up-type quark

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**Ashutosh Kumar Alok and Subhashish Banerjee**

*Indian Institute of Technology Jodhpur, Jodhpur 342011, India*

*E-mail:* akalok@iitj.ac.in, subhashish@iitj.ac.in

**Dinesh Kumar<sup>a,b</sup> and S. Uma Sankar<sup>a</sup>**

<sup>a</sup>*Indian Institute of Technology Bombay, Mumbai 400076, India*

<sup>b</sup>*Department of Physics, University of Rajasthan, Jaipur 302004, India*

*E-mail:* dinesh@phy.iitb.ac.in, uma@phy.iitb.ac.in

**David London**

*Physique des Particules, Université de Montréal,*

*C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7*

*E-mail:* london@lps.umontreal.ca

**ABSTRACT:** The VuQ model involves the addition of a vector isosinglet up-type quark to the standard model. In this model the full CKM quark mixing matrix is  $4 \times 3$ . Using present flavor-physics data, we perform a fit to this full CKM matrix, looking for signals of new physics (NP). We find that the VuQ model is very strongly constrained. There are no hints of NP in the CKM matrix, and any VuQ contributions to loop-level flavor-changing  $b \rightarrow s$ ,  $b \rightarrow d$  and  $s \rightarrow d$  transitions are very small. There can be significant enhancements of the branching ratios of the flavor-changing decays  $t \rightarrow uZ$  and  $t \rightarrow cZ$ , but these are still below present detection levels.

**KEYWORDS:** Vector-singlet up-type quark, Flavor physics, CKM matrix.

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## 1. Introduction

The standard model (SM) includes three generations of fermions. In particular, there are three down-type quarks ( $Q_{em} = -1/3$ :  $d$ ,  $s$ ,  $b$ ) and three up-type quarks ( $Q_{em} = 2/3$ :  $u$ ,  $c$ ,  $t$ ). All quarks with a given charge mix, so that there is a  $W$  coupling between each down-type and up-type quark. These couplings are tabulated in the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Now, there is no *a-priori* reason for there to be only three down-type and three up-type quarks. Indeed, many models of physics beyond the SM include new, exotic quarks. The simplest of these consider a fourth generation of quarks (denoted SM4), a vector isosinglet down-type quark  $b'$  (denoted VdQ; both  $b'_L$  and  $b'_R$  have weak isospin  $I = 0$ ), or a vector isosinglet up-type quark  $t'$  (denoted VuQ; both  $t'_L$  and  $t'_R$  have weak isospin  $I = 0$ ).

There are two distinct ways to look for signals of such new physics (NP). The first is via direct searches at colliders. To date, no signals of exotic quarks have been observed; the limits on the masses of these quarks depend on the assumptions about how they decay. Some recent results are (this is not exhaustive)  $m_{b',t'} > 685$  GeV (95% C.L.) for the SM4 model [1],  $m_{b'} \gtrsim 450$  GeV for the VdQ model [2], and  $m_{t'} > 687\text{-}782$  GeV (95% C.L.) for the VuQ model [3].

Second, one can look for indirect signals of the exotic quarks through their loop-level contributions to various processes. In fact, it is possible to simultaneously consider all such loop-level effects. This is done as follows. Most of these NP effects are charged-current interactions, which involve the CKM matrix. In the SM, the CKM matrix is  $3 \times 3$  and unitary. As such, it is parametrized by four parameters. However, in all NP models the full mixing matrix is larger than  $3 \times 3$ , so its parametrization requires additional parameters. The idea is then to perform a fit to the full CKM matrix using all the data. A signal of the NP will be the non-unitarity of the  $3 \times 3$  CKM matrix. That is, some of the NP parameters will be found to be nonzero.

At first glance, the analysis to search for NP is the same for all three models. First, in all cases the parametrization of the full CKM matrix has four SM and five NP parameters. Second, one uses the same flavor-physics data to perform a combined fit to these parameters. This yields the best-fit values of all the parameters, and indicates whether any of the NP parameters can be nonzero. However, the key point is that the contributions to the flavor-physics observables are model-dependent. That is, the effects on the observables vary from model to model, so that the analyses are *not* the same for the three models. The SM4 and VdQ models were examined in Refs. [4] and [10], respectively. In the present paper we consider the VuQ model [14, 15], in which the full CKM matrix is  $4 \times 3$ .

For the fit, in addition to the six directly-measured magnitudes of CKM matrix elements, we include flavor-physics observables that have small hadronic uncertain-

ties: (i)  $\epsilon_K$  from CP violation in  $K_L \rightarrow \pi\pi$ , (ii) the branching fractions of  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K_L \rightarrow \mu^+\mu^-$ , (iii)  $R_b$  and  $A_b$  from  $Z \rightarrow b\bar{b}$ , (iv)  $B_s^0$ - $\bar{B}_s^0$  and  $B_d^0$ - $\bar{B}_d^0$  mixing, (v) the time-dependent indirect CP asymmetries in  $B_d^0 \rightarrow J/\psi K_S$  and  $B_s^0 \rightarrow J/\psi \phi$ , (vi) the measurement of the CP-violating angle  $\gamma$  of the unitarity triangle from tree-level decays, (vii) the branching ratios of the inclusive decays  $B \rightarrow X_s l^+ l^-$  and  $B \rightarrow X_s \gamma$ , and of the exclusive decay  $B \rightarrow K\mu^+\mu^-$ , (viii) many observables in  $B \rightarrow K^*\mu^+\mu^-$ , (ix) the branching ratios of  $B_s^0 \rightarrow \mu^+\mu^-$ ,  $B_d^0 \rightarrow \mu^+\mu^-$  and  $B^+ \rightarrow \tau^+\nu_\tau$ , (x) the like-sign dimuon charge asymmetry  $A_{SL}^b$ , (xi) the oblique parameters  $S$  and  $T$ . The fit is carried out for  $m_{t'} = 800$  GeV and 1200 GeV.

In the VuQ model, the  $t'_L$  can mix with the  $u_L$ ,  $c_L$  and  $t_L$ . However, because the  $t'_L$  and  $\{u_L, c_L, t_L\}$  have different values of  $I_{3L}$  ( $I_{3L} = 0$  for  $t'_L$ ,  $I_{3L} = \frac{1}{2}$  for  $\{u_L, c_L, t_L\}$ ), this mixing will induce tree-level  $Z$ -mediated flavor-changing neutral currents (FCNC's) among the SM quarks. In particular, this means that  $D^0$ - $\bar{D}^0$  mixing occurs at tree level. Thus, in principle there can be constraints from the experimental measurement of this mixing. Now, in the SM, this mixing is due to a box diagram with internal  $d$ ,  $s$  and  $b$  quarks. The  $b$  contribution suffers a significant CKM suppression of  $O(\lambda^8)$ , so that  $D^0$ - $\bar{D}^0$  mixing is dominated by the contributions of the internal  $d$  and  $s$  quarks. Because these quarks are light, there can be large long-distance (LD) contributions to the mixing. At present, there is no definitive estimate of these LD effects. As a result, we do not have an accurate prediction of the value of  $D^0$ - $\bar{D}^0$  mixing within the SM, so that this measurement cannot be incorporated into the fit.

Once the fit has been performed, we can then make predictions for other quantities that are expected to be affected by the  $t'$  quark, while still being consistent with the above measurements. We examine the following observables: (i) the branching fraction of  $K_L \rightarrow \pi^0\nu\bar{\nu}$ , (ii) the branching fraction of  $B \rightarrow X_s\nu\bar{\nu}$ , (iii)  $D^0$ - $\bar{D}^0$  mixing and the branching fraction of  $D^0 \rightarrow \mu^+\mu^-$ , and (iv) the branching fraction of  $t \rightarrow qZ$  ( $q = u, c$ ).

The paper is organized as follows. In Sec. 2, we define the CKM parametrization and discuss the measurements used in the  $\chi^2$  fit. The results of the fit are presented in Sec. 3. Given these results, we calculate the possible effects of the VuQ model on several other flavor observables in Sec. 4. Sec. 5 summarizes the results.

## 2. Constraints on the CKM Matrix

In the VdQ model the CKM matrix is  $3 \times 4$ . It was shown in Ref. [16] that this is the upper  $3 \times 4$  submatrix of the  $4 \times 4$  SM4 CKM matrix, denoted CKM4. Now, there are many parametrizations of CKM4. For the VdQ model, it is best to choose one in which the new matrix elements  $V_{ub'}$ ,  $V_{cb'}$  and  $V_{tb'}$  take simple forms. With this in mind, the Dighe-Kim parametrization of Refs. [16, 17] was used in Ref. [10].

The logic is similar for the VuQ model. In this model the CKM matrix is  $4 \times 3$ :

$$V_{\text{VuQ}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{t'd} & V_{t's} & V_{t'b} \end{pmatrix}. \quad (2.1)$$

$V_{\text{VuQ}}$  is the left-hand  $4 \times 3$  submatrix of CKM4. Here it is best to choose a parametrization of CKM4 in which the new matrix elements  $V_{t'd}$ ,  $V_{t's}$  and  $V_{t'b}$  take simple forms. We use the Hou-Soni-Steger parametrization [18, 19]. Here,

$$\begin{aligned} V_{us} &\equiv \lambda, & V_{cb} &\equiv A\lambda^2, & V_{ub} &\equiv A\lambda^3 C e^{-i\delta_{ub}}, \\ V_{t'd} &\equiv -P\lambda^3 e^{i\delta_{t'd}}, & V_{t's} &\equiv -Q\lambda^2 e^{i\delta_{t's}}, & V_{t'b} &\equiv -r\lambda, \end{aligned} \quad (2.2)$$

where  $\lambda$  is the sine of the Cabibbo angle. There are four SM parameters ( $\lambda$ ,  $A$ ,  $C$ ,  $\delta_{ub}$ ) and five NP parameters ( $P$ ,  $Q$ ,  $r$ ,  $\delta_{t'd}$ ,  $\delta_{t's}$ ). Of the remaining six CKM matrix elements,  $V_{ud}$ ,  $V_{cd}$  and  $V_{cs}$  retain their SM parametrizations:

$$V_{ud} = 1 - \frac{\lambda^2}{2}, \quad V_{cd} = -\lambda, \quad V_{cs} = 1 - \frac{\lambda^2}{2}, \quad (2.3)$$

but  $V_{td}$ ,  $V_{ts}$  and  $V_{tb}$  are modified:

$$\begin{aligned} V_{td} &= A\lambda^3 (1 - C e^{i\delta_{ub}}) - P r \lambda^4 e^{i\delta_{t'd}} + \frac{1}{2} A C \lambda^5 e^{i\delta_{ub}}, \\ V_{ts} &= -A\lambda^2 - Q r \lambda^3 e^{i\delta_{t's}} + A \lambda^4 \left( \frac{1}{2} - C e^{i\delta_{ub}} \right), \\ V_{tb} &= 1 - \frac{1}{2} r^2 \lambda^2. \end{aligned} \quad (2.4)$$

In the limit  $P = Q = r = 0$ , only the elements present in the  $3 \times 3$  CKM matrix retain nontrivial values, and the above expansion corresponds to the Wolfenstein parametrization [20] with  $C = \sqrt{\rho^2 + \eta^2}$  and  $\delta_{ub} = \tan^{-1}(\eta/\rho)$ . In this limit,  $V_{tb} = 1$ . In the VuQ model,  $r$  can be nonzero, leading to a deviation of  $V_{tb}$  from 1.

For the fit, we consider all observables that can constrain the parameters of the CKM matrix. The total  $\chi^2$  is written as a function of these parameters, and their best-fit values are those that minimize this  $\chi^2$  function. The total  $\chi^2$  function is defined as

$$\begin{aligned} \chi_{\text{total}}^2 &= \chi_{\text{CKM}}^2 + \chi_{|\epsilon_K|}^2 + \chi_{K \rightarrow \pi^+ \nu \bar{\nu}}^2 + \chi_{K_L \rightarrow \mu^+ \mu^-}^2 + \chi_{Z \rightarrow b\bar{b}}^2 + \chi_{B_d^0}^2 + \chi_{M_R}^2 + \chi_{\sin 2\beta}^2 \\ &+ \chi_{\sin 2\beta_s}^2 + \chi_\gamma^2 + \chi_{B \rightarrow X_s l^+ l^-}^2 + \chi_{B \rightarrow X_s \gamma}^2 + \chi_{B \rightarrow K \mu^+ \mu^-}^2 + \chi_{B \rightarrow K^* \mu^+ \mu^-}^2 \\ &+ \chi_{B^+ \rightarrow \pi^+ \mu^+ \mu^-}^2 + \chi_{B_q \rightarrow \mu^+ \mu^-}^2 + \chi_{B \rightarrow \tau \nu}^2 + \chi_{A_{SL}^b}^2 + \chi_{\text{Oblique}}^2. \end{aligned} \quad (2.5)$$

In our analysis, the  $\chi^2$  of an observable  $A$  whose measured value is  $(A_{\text{exp}}^c \pm A_{\text{exp}}^{\text{err}})$  is defined as

$$\chi_A^2 = \left( \frac{A - A_{\text{exp}}^c}{A_{\text{exp}}^{\text{err}}} \right)^2. \quad (2.6)$$

$ V_{ud}  = 0.97425 \pm 0.00022$	$\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{low}} = (1.60 \pm 0.48) \times 10^{-6}$ [22]
$ V_{us}  = 0.2252 \pm 0.0009$	$\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{high}} = (0.57 \pm 0.16) \times 10^{-6}$ [22]
$ V_{cd}  = 0.230 \pm 0.011$	$10^9 \text{ GeV}^2 \times \langle \frac{d\mathcal{B}}{dq^2} \rangle (B \rightarrow K \mu^+ \mu^-)_{\text{low}} = 18.7 \pm 3.6$ [23]
$ V_{cs}  = 1.006 \pm 0.023$	$10^9 \text{ GeV}^2 \times \langle \frac{d\mathcal{B}}{dq^2} \rangle (B \rightarrow K \mu^+ \mu^-)_{\text{high}} = 9.5 \pm 1.7$ [23]
$ V_{ub}  = 0.00382 \pm 0.00021$	$\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-) = (2.60 \pm 0.61) \times 10^{-8}$ [24]
$ V_{cb}  = (40.9 \pm 1.0) \times 10^{-3}$	$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.7 \pm 1.1) \times 10^{-10}$
$\gamma = (68.0 \pm 11.0)^\circ$	$\mathcal{B}(K_L \rightarrow \mu^+ \mu^-) = (0 \pm 1.56) \times 10^{-9}$ [25]
$ \epsilon_k  \times 10^3 = 2.228 \pm 0.011$	$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) = (2.9 \pm 0.7) \times 10^{-9}$ [26, 27, 28]
$\Delta M_d = (0.507 \pm 0.004) \text{ ps}^{-1}$ [21]	$\mathcal{B}(B_d \rightarrow \mu^+ \mu^-) = (3.9 \pm 1.6) \times 10^{-10}$ [26, 27, 28]
$\Delta M_s = (17.72 \pm 0.04) \text{ ps}^{-1}$ [21]	$\mathcal{B}(B \rightarrow X_s \gamma) = (3.55 \pm 0.26) \times 10^{-4}$
$S_{J/\psi \phi} = 0.00 \pm 0.07$ [21]	$\mathcal{B}(B \rightarrow \tau \bar{\nu}) = (1.14 \pm 0.22) \times 10^{-4}$ [21]
$S_{J/\psi K_S} = 0.68 \pm 0.02$ [21]	$A_{sl}^b = (-4.96 \pm 1.69) \times 10^{-3}$ [29]
$S = 0.00 \pm 0.11$	$A_b = 0.923 \pm 0.020$ [30]
$T = 0.02 \pm 0.12$	$R_b = 0.2164 \pm 0.0007$ [30]

Table 1: Experimental values of flavor-physics observables used as constraints. For  $V_{ub}$  we use the weighted average from the inclusive and exclusive semileptonic decays,  $V_{ub}^{inc} = (44.1 \pm 3.1) \times 10^{-4}$  and  $V_{ub}^{exc} = (32.3 \pm 3.1) \times 10^{-4}$ . When not explicitly stated, we take the inputs from the Particle Data Group [31]. Wherever there are asymmetric experimental errors, they are symmetrized by taking the largest side error. Also, wherever there is more than one source of uncertainty, the total error is obtained by adding these in quadrature.

In the following subsections, we discuss the various experimental measurements used in the fit, and give their individual contributions to  $\chi^2_{\text{total}}$ .

The current experimental values for the 68 flavor-physics observables enumerated in the introduction are listed in Tables 1 and 2. The theoretical expressions for these observables require additional inputs in the form of decay constants, bag parameters, QCD corrections and other parameters. These are listed in Table 3.

## 2.1 Direct measurements of the CKM elements

The latest values for the direct measurements of the magnitudes of the CKM matrix elements can be found in Ref. [31]. The contribution to  $\chi^2_{\text{total}}$  from these measurements is given by

$$\begin{aligned} \chi^2_{\text{CKM}} = & \left( \frac{|V_{us}| - 0.2252}{0.0009} \right)^2 + \left( \frac{|V_{ud}| - 0.97425}{0.00022} \right)^2 + \left( \frac{|V_{cs}| - 1.006}{0.023} \right)^2 \\ & + \left( \frac{|V_{cd}| - 0.230}{0.011} \right)^2 + \left( \frac{|V_{ub}| - 0.00382}{0.00021} \right)^2 + \left( \frac{|V_{cb}| - 0.0409}{0.001} \right)^2. \end{aligned} \quad (2.7)$$

## 2.2 CP violation in $K_L \rightarrow \pi\pi$ : $\epsilon_K$

In the VuQ model, the mixing amplitude  $M_K^{12}$  is modified due to an additional con-

$q^2 = 0.1\text{-}2 \text{ GeV}^2$	$q^2 = 2\text{-}4.3 \text{ GeV}^2$	$q^2 = 4.3\text{-}8.68 \text{ GeV}^2$
$\langle \frac{d\mathcal{B}}{dq^2} \rangle = (0.60 \pm 0.10) \times 10^{-7}$	$\langle \frac{d\mathcal{B}}{dq^2} \rangle = (0.30 \pm 0.05) \times 10^{-7}$	$\langle \frac{d\mathcal{B}}{dq^2} \rangle = (0.49 \pm 0.08) \times 10^{-7}$
$\langle F_L \rangle = 0.37 \pm 0.11$	$\langle F_L \rangle = 0.74 \pm 0.10$	$\langle F_L \rangle = 0.57 \pm 0.08$
$\langle P_1 \rangle = -0.19 \pm 0.40$	$\langle P_1 \rangle = -0.29 \pm 0.65$	$\langle P_1 \rangle = 0.36 \pm 0.31$
$\langle P_2 \rangle = 0.03 \pm 0.15$	$\langle P_2 \rangle = 0.50 \pm 0.08$	$\langle P_2 \rangle = -0.25 \pm 0.08$
$\langle P'_4 \rangle = 0.00 \pm 0.52$	$\langle P'_4 \rangle = 0.74 \pm 0.60$	$\langle P'_4 \rangle = 1.18 \pm 0.32$
$\langle P'_5 \rangle = 0.45 \pm 0.24$	$\langle P'_5 \rangle = 0.29 \pm 0.40$	$\langle P'_5 \rangle = -0.19 \pm 0.16$
$\langle P'_6 \rangle = 0.24 \pm 0.23$	$\langle P'_6 \rangle = -0.15 \pm 0.38$	$\langle P'_6 \rangle = 0.04 \pm 0.16$
$\langle P'_8 \rangle = -0.12 \pm 0.56$	$\langle P'_8 \rangle = -0.3 \pm 0.60$	$\langle P'_8 \rangle = 0.58 \pm 0.38$
$q^2 = 14.18\text{-}16 \text{ GeV}^2$	$q^2 = 16\text{-}19 \text{ GeV}^2$	
$\langle \frac{d\mathcal{B}}{dq^2} \rangle = (0.56 \pm 0.10) \times 10^{-7}$	$\langle \frac{d\mathcal{B}}{dq^2} \rangle = (0.41 \pm 0.07) \times 10^{-7}$	
$\langle F_L \rangle = 0.33 \pm 0.09$	$\langle F_L \rangle = 0.38 \pm 0.09$	
$\langle P_1 \rangle = 0.07 \pm 0.28$	$\langle P_1 \rangle = -0.71 \pm 0.36$	
$\langle P_2 \rangle = -0.50 \pm 0.03$	$\langle P_2 \rangle = -0.32 \pm 0.08$	
$\langle P'_4 \rangle = -0.18 \pm 0.70$	$\langle P'_4 \rangle = 0.70 \pm 0.52$	
$\langle P'_5 \rangle = -0.79 \pm 0.27$	$\langle P'_5 \rangle = -0.60 \pm 0.21$	
$\langle P'_6 \rangle = 0.18 \pm 0.25$	$\langle P'_6 \rangle = -0.31 \pm 0.39$	
$\langle P'_8 \rangle = -0.40 \pm 0.60$	$\langle P'_8 \rangle = 0.12 \pm 0.54$	

Table 2: Experimental values of the observables in  $B \rightarrow K^* \mu^+ \mu^-$  used as constraints. These are taken from Refs. [32, 33]. Here the errors have been symmetrized by taking the largest side error. Also, wherever there is more than one source of uncertainty, the total error is obtained by adding these in quadrature.

tribution coming from a virtual  $t'$  quark in the box diagram. There is a sizeable LD contribution to the mass difference  $\Delta M_K$  in the  $K$  system, for which, at present, there is no definitive estimate. We therefore do not include  $\Delta M_K$  in our analysis. However,  $|\epsilon_K|$ , the parameter describing the mixing-induced  $CP$  asymmetry in neutral  $K$  decays, and which is proportional to  $\text{Im}(M_K^{12})$ , is theoretically clean and is a well-measured quantity. The theoretical expression for  $|\epsilon_K|$  in the presence of a  $t'$  quark is given in Refs. [4, 6].

To calculate the contribution of  $|\epsilon_K|$  to  $\chi_{\text{total}}^2$ , we use the quantity

$$K_{\text{mix}} = \frac{12\sqrt{2}\pi^2(\Delta M_K)_{\text{exp}}|\epsilon_K|}{G_F^2 M_W^2 f_K^2 M_K \hat{B}_K k_\epsilon} - \text{Im} [\eta_c (V_{cs} V_{cd}^*)^2 S(x_c)] . \quad (2.8)$$

With the experimental and theoretical inputs given in Tables 1 and 3, we find

$$K_{\text{mix, exp}} = (1.69 \pm 0.05) \times 10^{-7} . \quad (2.9)$$

The QCD correction  $\eta_{ct}$  appears in the theoretical expression of  $|\epsilon_K|$ . In order to take its error into account, we consider it to be a parameter and have added a contribution

$G_F = 1.16637 \times 10^{-5} \text{ Gev}^{-2}$	$\tau_{B_s} = (1.497 \pm 0.026) \text{ ps}$
$\sin^2 \theta_w = 0.23116$	$\tau_{B^\pm} = (1.641 \pm 0.008) \text{ ps}$
$\alpha(M_Z) = \frac{1}{127.9}$	$\eta_t = 0.5765 [34]$
$\alpha_s(M_Z) = 0.1184$	$\eta_{ct} = 0.496 \pm 0.047 [35]$
$m_t(m_t) = 163 \text{ GeV}$	$f_K = 0.1561 \pm 0.0011 [36]$
$m_c(m_c) = 1.275 \pm 0.025 \text{ GeV}$	$B_K = 0.767 \pm 0.010 [36]$
$m_b(m_b) = 4.18 \pm 0.03 \text{ GeV}$	$\Delta M_K = (0.5292 \pm 0.0009) \times 10^{-2} \text{ ps}^{-1}$
$M_W = 80.385 \text{ GeV}$	$\kappa_\epsilon = 0.94 \pm 0.02 [37, 38]$
$M_Z = 91.1876 \text{ GeV}$	$\kappa_+ = (5.36 \pm 0.026) \times 10^{-11} [39]$
$M_K = 0.497614 \text{ GeV}$	$\kappa_\mu = (2.009 \pm 0.017) \times 10^{-9} [40]$
$M_{K^*} = 0.89594 \text{ GeV}$	$f_{bd} = (190.5 \pm 4.2) \text{ MeV} [41]$
$M_D = 1.86486 \text{ GeV}$	$f_{bs} = (227.7 \pm 4.5) \text{ MeV} [41]$
$M_{B_d} = 5.27917 \text{ GeV}$	$f_{B_d^0} \sqrt{B_{B_d^0}} = (0.216 \pm 0.015) \text{ GeV} [41]$
$M_{B_s} = 5.36677 \text{ GeV}$	$\xi = 1.268 \pm 0.063 [41]$
$M_{B^\pm} = 5.27926 \text{ GeV}$	$\mathcal{B}(B \rightarrow X_c \ell \nu) = (10.61 \pm 0.17) \times 10^{-2}$
$m_\mu = 0.105 \text{ GeV}$	$m_c/m_b = 0.29 \pm 0.02$
$m_\tau = 1.77682 \text{ GeV}$	
$\tau_{B_d} = (1.519 \pm 0.007) \text{ ps}$	

Table 3: Decay constants, bag parameters, QCD corrections and other parameters used in our analysis. When not explicitly stated, we take the inputs from the Particle Data Group [31].

to  $\chi_{\text{total}}^2$ . We hold the other QCD correction  $\eta_t$  fixed to its central value because its error is very small. The total contribution to  $\chi_{\text{total}}^2$  from  $|\epsilon_K|$  is then

$$\chi_{|\epsilon_K|}^2 = \left( \frac{K_{\text{mix}} - 1.69 \times 10^{-7}}{0.05 \times 10^{-7}} \right)^2 + \left( \frac{\eta_{ct} - 0.496}{0.047} \right)^2. \quad (2.10)$$

### 2.3 Branching fraction of the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

In Refs. [42, 43], it was shown that the LD contribution to  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  is suppressed – it is three orders of magnitude smaller than the short-distance (SD) contribution. The SM prediction for this observable is therefore under good control. The decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  occurs via loops containing virtual heavy particles, and hence is sensitive to the  $t'$  quark. The theoretical expression for  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  in the presence of a  $t'$  quark is given in Refs. [4, 6].

With the inputs given in Tables 1 and 3, we estimate

$$\frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\kappa_+} = 3.17 \pm 2.05, \quad (2.11)$$

where

$$\kappa_+ = r_{K^+} \frac{3\alpha^2 \mathcal{B}(K^+ \rightarrow \pi^0 e^+ \nu)}{2\pi^2 \sin^4 \theta_W} \lambda^8 . \quad (2.12)$$

Here  $r_{K^+} = 0.901$  encapsulates the isospin-breaking corrections in relating the branching ratio of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  to that of the well-measured decay  $K^+ \rightarrow \pi^0 e^+ \nu$ .

In order to include  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  in the fit, we define

$$\chi^2_{K^+ \rightarrow \pi^+ \nu \bar{\nu}} = \left( \frac{[\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})/\kappa_+] - 3.17}{2.05} \right)^2 . \quad (2.13)$$

## 2.4 Branching fraction of the decay $K_L \rightarrow \mu^+ \mu^-$

Unlike  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , the decay  $K_L \rightarrow \mu^+ \mu^-$  is not cleanly dominated by the SD contribution. However, it is possible to estimate the LD contribution to this decay. The absorptive LD contribution is estimated using  $K_L \rightarrow \gamma\gamma$ , while the dispersive LD contribution is estimated using chiral perturbation theory along with the experimental inputs on various  $K$  decays. Due to uncertainties involved in the extraction of the dispersive contribution, one can only obtain a conservative upper limit on the SD contribution to  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)$ , which is  $\leq 2.5 \times 10^{-9}$  [25]. With all the inputs given in Tables 1 and 3, we estimate

$$\frac{\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)}{\kappa_\mu} = 0 \pm 0.778 , \quad (2.14)$$

where

$$\kappa_\mu = \frac{\alpha^2 \mathcal{B}(K^+ \rightarrow \mu^+ \nu_\mu)}{\pi^2 \sin^4 \theta_W} \frac{\tau(K_L)}{\tau(K^+)} \lambda^8 . \quad (2.15)$$

In the VuQ model, the theoretical expression for  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)/\kappa_\mu$  is given by

$$\begin{aligned} \frac{\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)}{\kappa_\mu} = & \left( \frac{\text{Re}(V_{cd} V_{cs}^*)}{\lambda} P_c + \frac{\text{Re}(V_{td} V_{ts}^*)}{\lambda^5} Y(x_t) \right. \\ & \left. + \frac{\text{Re}(V_{t'd} V_{t's}^*)}{\lambda^5} Y(x_{t'}) \right)^2 . \end{aligned} \quad (2.16)$$

Here  $Y(x)$  is the structure function in the  $t$  or  $t'$  sector [44, 45], while  $P_c$  is the corresponding structure function in the charm sector. Its NNLO QCD-corrected value is  $P_c = 0.115 \pm 0.018$  [40]. In order to include  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)$  in the fit, we define

$$\chi^2_{K_L \rightarrow \mu^+ \mu^-} = \left( \frac{\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)/\kappa_\mu - 0}{0.778} \right)^2 + \left( \frac{P_c - 0.115}{0.018} \right)^2 . \quad (2.17)$$

Thus, the error on  $P_c$  has been taken into account by considering it to be a parameter and adding a contribution to  $\chi^2_{\text{total}}$ .

## 2.5 $Z \rightarrow b\bar{b}$ decay

Here we include constraints from  $R_b$  and  $A_b$ , respectively the vertex correction and forward-backward asymmetry in  $Z \rightarrow b\bar{b}$ . The theoretical expressions for  $R_b$  and  $A_b$  in the VuQ model are given in Ref. [14]. We have

$$\chi_{Z \rightarrow bb}^2 = \left( \frac{R_b - 0.216}{0.001} \right)^2 + \left( \frac{A_b - 0.923}{0.020} \right)^2. \quad (2.18)$$

## 2.6 $B_q^0$ - $\bar{B}_q^0$ mixing ( $q = d, s$ )

The theoretical expressions for  $M_{12}^q$  ( $q = d, s$ ) in the presence of a  $t'$  quark, which then lead to  $\Delta M_d$  and  $\Delta M_s$ , are given in Refs. [4, 6]. To calculate  $\chi_{B_d^0}^2$  for  $B_d^0$ - $\bar{B}_d^0$  mixing, we use the quantity

$$B_{\text{mix}}^d = \frac{6\pi^2 \Delta M_d}{G_F^2 M_W^2 M_{B_d} \hat{B}_{bd} f_{B_d^0}^2}. \quad (2.19)$$

With the inputs given in Table 1, we get

$$B_{\text{mix,exp}}^d = (9.12249 \pm 1.26905) \times 10^{-5}, \quad (2.20)$$

leading to

$$\chi_{B_d^0}^2 = \left( \frac{B_{\text{mix}}^d - 9.12249 \times 10^{-5}}{1.26905 \times 10^{-5}} \right)^2. \quad (2.21)$$

To take  $B_s^0$ - $\bar{B}_s^0$  mixing into account, we define

$$M_R = \frac{\Delta M_s}{\Delta M_d} \frac{M_{B_d}}{M_{B_s}} \frac{1}{\xi^2}, \quad (2.22)$$

whose measured value is

$$M_{R,\text{exp}} = 21.3831 \pm 2.1321. \quad (2.23)$$

Then

$$\chi_{M_R}^2 = \left( \frac{M_R - 21.3831}{2.1321} \right)^2. \quad (2.24)$$

## 2.7 Indirect CP violation in $B_d^0 \rightarrow J/\psi K_S$ and $B_s^0 \rightarrow J/\psi \phi$

The theoretical expressions for  $M_{12}^q$  ( $q = d, s$ ) in the VuQ model are discussed in the previous subsection. In the SM, indirect CP violation in  $B_d^0 \rightarrow J/\psi K_S$  and  $B_s^0 \rightarrow J/\psi \phi$  probes  $\sin 2\beta$  and  $\sin 2\beta_s$ , respectively. With NP, we have

$$S_{J/\psi K_S} = \frac{\text{Im}(M_{12}^d)}{M_{12}^d}, \quad S_{J/\psi \phi} = -\frac{\text{Im}(M_{12}^s)}{M_{12}^s}. \quad (2.25)$$

The experimentally-measured values of  $\sin 2\beta$  and  $\sin 2\beta_s$  are given in Ref. [31]. Then

$$\chi_{\sin 2\beta}^2 = \left( \frac{S_{J/\psi K_S} - 0.68}{0.02} \right)^2, \quad \chi_{\sin 2\beta_s}^2 = \left( \frac{S_{J/\psi \phi} - 0.00}{0.07} \right)^2. \quad (2.26)$$

## 2.8 CKM angle $\gamma$

In the Wolfenstein parametrization, the CKM angle  $\gamma = \tan^{-1}(\eta/\rho)$ , which is the argument of  $V_{ub}$ . As this angle is measured in tree-level decays, its value is unchanged with the addition of a vector isosinglet up-type quark. Therefore the  $\chi^2$  of  $\gamma$  is given by

$$\chi_\gamma^2 = \left( \frac{\delta_{ub} - 68 (\pi/180)}{11 (\pi/180)} \right)^2. \quad (2.27)$$

## 2.9 Branching ratio of $B \rightarrow X_s l^+ l^-$ ( $l = e, \mu$ )

The quark-level transition  $b \rightarrow s l^+ l^-$  can occur only at loop level within the SM, so that it can be used to test higher-order corrections to the SM, and to constrain various NP models. Within the SM, the effective Hamiltonian for this transition can be written as

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts} V_{tb}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu), \quad (2.28)$$

where the form of the operators  $O_i$  and the expressions for calculating the coefficients  $C_i$  are given in Ref. [46]. In the VuQ model only the values of the Wilson coefficients  $C_{7,8,9,10}$  are changed via the virtual exchange of the  $t'$  quark. The modified Wilson coefficients in the vector-singlet up-quark model can then be written as [4, 6]

$$C_j^{\text{tot}}(\mu_b) = C_j(\mu_b) + \frac{V_{t's} V_{t'b}^*}{V_{ts} V_{tb}^*} C_j^{t'}(\mu_b), \quad (2.29)$$

where  $j = 7, 8, 9, 10$ . The new wilson coefficients  $C_j^{t'}$  can be calculated from the expression of  $C_j$  by replacing  $m_t$  by  $m_{t'}$ .

The inclusive decay mode  $B \rightarrow X_s l^+ l^-$  has relatively small theoretical errors as compared to the exclusive decay modes  $B \rightarrow (K, K^*) l^+ l^-$ . However, the inclusive decays are less readily accessible experimentally. The branching ratio of  $B \rightarrow X_s l^+ l^-$  has been measured by the Belle and BaBar Collaborations using the sum-of-exclusive technique. The latest Belle measurement uses only 25% of its final data set [47]. The BaBar Collaboration has recently published the measurement of  $\mathcal{B}(B \rightarrow X_s l^+ l^-)$  using the full data set, which corresponds to  $471 \times 10^6 B\bar{B}$  events [22]. This is an update of their previous result, which was based on a data sample of  $89 \times 10^6 B\bar{B}$  events [48].

The prediction for the branching ratio is relatively cleaner in the low- $q^2$  ( $1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$ ) and high- $q^2$  ( $14.2 \text{ GeV}^2 \leq q^2 \leq m_b^2$ ) regions. We consider both regions in the fit. The theoretical predictions for  $\mathcal{B}(B \rightarrow X_s l^+ l^-)$  are computed using the program **SuperIso** [49, 50], in which the higher-order and power corrections are implemented following Refs. [51, 52], while the electromagnetic logarithmically-enhanced corrections are taken from Refs. [53]. Bremsstrahlung contributions are implemented following Refs. [54].

The contribution to  $\chi^2_{\text{total}}$  is

$$\begin{aligned}\chi^2_{B \rightarrow X_s l^+ l^-} &= \left( \frac{\mathcal{B}(B \rightarrow X_s l^+ l^-)_{\text{low}} - 1.6 \times 10^{-6}}{0.49 \times 10^{-6}} \right)^2 \\ &\quad + \left( \frac{\mathcal{B}(B \rightarrow X_s l^+ l^-)_{\text{high}} - 0.57 \times 10^{-6}}{0.23 \times 10^{-6}} \right)^2,\end{aligned}\quad (2.30)$$

where we have added a theoretical error of 7% to  $\mathcal{B}(B \rightarrow X_s l^+ l^-)_{\text{low}}$ , which includes corrections due to the renormalization scale and quark masses, and a theoretical error of 30% to  $\mathcal{B}(B \rightarrow X_s l^+ l^-)_{\text{high}}$ , which includes the non-perturbative QCD corrections.

## 2.10 Branching ratio of $B \rightarrow X_s \gamma$

The quantity we use for  $B \rightarrow X_s \gamma$  is

$$\tilde{R} = \frac{\pi f(\hat{m}_c) \kappa(\hat{m}_c)}{6 \alpha} \frac{\mathcal{B}(B \rightarrow X_s \gamma)}{\mathcal{B}(B \rightarrow X_c e \bar{\nu}_e)}, \quad (2.31)$$

where the ratio of the two branching fractions is taken in order to reduce the large uncertainties arising from  $b$ -quark mass. The theoretical expression for  $\mathcal{B}(B \rightarrow X_s \gamma)$  is given in Refs. [4, 6]. From this, one can deduce the expression for  $\tilde{R}$ . The measured value of  $\tilde{R}$  is

$$\tilde{R}_{\text{exp}} = 0.1069 \pm 0.0120, \quad (2.32)$$

where we have added an overall correction of 5% due to the non-perturbative terms. The contribution to  $\chi^2_{\text{total}}$  is

$$\chi^2_{B \rightarrow X_s \gamma} = \left( \frac{\tilde{R} - 0.1069}{0.0120} \right)^2. \quad (2.33)$$

## 2.11 Branching ratio of $B \rightarrow K \mu^+ \mu^-$

The theoretical expression for  $\langle d\mathcal{B}/dq^2 \rangle (B \rightarrow K \mu^+ \mu^-)$  in the SM is given in Refs. [55, 56], and can be adapted straightforwardly to the VuQ model. The predictions for the branching ratio are relatively cleaner in the low- $q^2$  ( $1.1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$ ) and the high- $q^2$  ( $15 \text{ GeV}^2 \leq q^2 \leq 22 \text{ GeV}^2$ ) regions. Here, we consider both regions in the fit. We use the recent LHCb measurements of  $\langle d\mathcal{B}/dq^2 \rangle (B \rightarrow K \mu^+ \mu^-)$  [23].

Our analysis of  $B \rightarrow K \mu^+ \mu^-$  in the low- $q^2$  region is based on QCD factorization (QCdf) [57]. The factorizable and non-factorizable corrections of  $O(\alpha_s)$  are included in our numerical analysis following Refs. [55, 57]. In the high- $q^2$  region, following Ref. [56], we use the improved Isgur-Wise relation between the form factors which are determined using light-cone QCD sum-rule calculations extrapolated to the high- $q^2$  region. The contribution to  $\chi^2_{\text{total}}$  from  $B \rightarrow K \mu^+ \mu^-$  is

$$\begin{aligned}\chi^2_{B \rightarrow K \mu^+ \mu^-} &= \left( \frac{\langle \frac{d\mathcal{B}}{dq^2} \rangle (B \rightarrow K \mu^+ \mu^-)_{\text{low}} - 18.7 \times 10^{-9}}{6.67 \times 10^{-9}} \right)^2 \\ &\quad + \left( \frac{\langle \frac{d\mathcal{B}}{dq^2} \rangle (B \rightarrow K \mu^+ \mu^-)_{\text{high}} - 9.5 \times 10^{-9}}{3.32 \times 10^{-9}} \right)^2,\end{aligned}\quad (2.34)$$

where, following Refs. [55, 56], we have included a theoretical error of 30% in both low- and high- $q^2$  bins. This is due mainly to uncertainties in the  $B \rightarrow K$  form factors.

## 2.12 Constraints from $B \rightarrow K^* \mu^+ \mu^-$

The recent LHCb measurements of new angular observables in  $B \rightarrow K^* \mu^+ \mu^-$  exhibit small tensions with the SM predictions [33, 58]. These tensions can be due to NP, but can also be attributed to underestimated hadronic power corrections, or can simply be a statistical fluctuation. In our analysis, we include all measured observables in  $B \rightarrow K^* \mu^+ \mu^-$  in the low- and high- $q^2$  regions. The experimental results for  $B \rightarrow K^* \mu^+ \mu^-$  decay are given in Table 2, and are taken from Refs. [32, 33].

The complete angular distribution for the decay  $B \rightarrow K^* \mu^+ \mu^-$  is described by four independent kinematic variables: the lepton-pair invariant mass squared  $q^2$ , two polar angles  $\theta_\mu$  and  $\theta_K$ , and the angle between the planes of the dimuon and  $K\pi$  decays,  $\phi$ . The differential decay distribution of  $B \rightarrow K^* \mu^+ \mu^-$  can be written as

$$\frac{d^4\Gamma[B \rightarrow K^*(\rightarrow K\pi)\mu^+\mu^-]}{dq^2 d\cos\theta_l d\cos\theta_K d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi). \quad (2.35)$$

where the angular-dependent term can be written as

$$\begin{aligned} J(q^2, \theta_l, \theta_K, \phi) = & J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_l \\ & + J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi \\ & + J_5 \sin 2\theta_K \sin \theta_l \cos \phi + (J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_l \\ & + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi. \end{aligned} \quad (2.36)$$

For massless leptons, the  $J_i$ 's depend on the six complex  $K^*$  spin amplitudes  $A_{||}^{L,R}$ ,  $A_{\perp}^{L,R}$  and  $A_0^{L,R}$ . For example,

$$J_{1s} = \frac{3}{4} [|A_{\perp}^L|^2 + |A_{||}^L|^2 + |A_{\perp}^R|^2 + |A_{||}^R|^2]. \quad (2.37)$$

For massive leptons, the additional amplitude  $A_t$  has to be introduced. In our analysis, the muon mass is included.

The analysis of  $B \rightarrow K^* \mu^+ \mu^-$  in the low- $q^2$  region is based on QCdf [57] and its quantum field-theoretical formulation, Soft-Collinear Effective Theory (SCET). In the limits of a heavy  $b$  quark and an energetic  $K^*$  meson [59, 60, 61], the form factors can be expanded in the small ratios  $\Lambda_{\text{QCD}}/m_b$  and  $\Lambda_{\text{QCD}}/E$ , where  $E$  is the energy of the  $K^*$  meson. At leading order in  $1/m_b$  and  $\alpha_s$ , the seven *a-priori* independent  $B \rightarrow K^*$  form factors reduce to two universal form factors  $\xi_{\perp,||}$  [59, 60, 61, 62, 63]. The symmetry-breaking corrections of  $O(\alpha_s)$ , both factorizable and non-factorizable, are included in our numerical analysis following Ref. [57]. Regarding the  $\Lambda_{\text{QCD}}/m_b$  corrections to the QCdf amplitudes, we do not have any means to calculate them in

general. These power corrections can only be estimated by combining QCDF/SCET results with a QCD sum rule approach, see Refs. [64, 65].

The analysis of  $B \rightarrow K^* \mu^+ \mu^-$  in the high- $q^2$  region is based on the heavy-quark effective theory framework by Grinstein and Pirjol [66]. It was shown in Refs. [66, 67] that an operator product expansion is applicable, which allows one to obtain the  $B \rightarrow K^* \mu^+ \mu^-$  matrix elements in a systematic expansion in  $\alpha_s$  and in  $\Lambda_{\text{QCD}}/m_b$ . The leading  $\Lambda_{\text{QCD}}/m_b$  corrections are parametrically suppressed and contribute only at the few percent level. The improved Isgur-Wise relations between the form factors at leading order in  $1/m_b$  lead to simple expressions for the  $K^*$  spin amplitudes to leading order in  $1/m_b$  [68, 69, 70]. For the form factors in the high- $q^2$  region, we have used the recent lattice results [71, 72].

Of course, these theoretical predictions have errors associated with them [65, 68, 73, 74, 75, 76, 77]. The main sources of uncertainties in the low- $q^2$  region, excluding uncertainties due to CKM matrix elements, are (i) the form factors, (ii) the unknown  $1/m_b$  subleading corrections, (iii) the quark masses, and (iv) the renormalization scale  $\mu_b$ . In the high- $q^2$  region, there is an additional subleading correction of  $O(1/m_b)$  to the improved Isgur-Wise form factor relations. For each  $B \rightarrow K^* \mu^+ \mu^-$  observable  $O_j$ , the theoretical error is incorporated in the fit by multiplying the theoretical result by  $(1 \pm X_j)$ , where  $X_j$  is the total theoretical error corresponding to the  $j^{\text{th}}$  observable and can be easily estimated using Table II of Ref. [73].

For  $B \rightarrow K^* \mu^+ \mu^-$ , we use the observables  $\langle d\mathcal{B}/dq^2 \rangle$ ,  $P_1$ ,  $P_2$ ,  $P'_4$ ,  $P'_5$ ,  $P'_6$ ,  $P'_8$  and  $F_L$  in the low- $q^2$  bins 0.1-2 GeV $^2$ , 2.0-4.3 GeV $^2$ , 4.3-8.68 GeV $^2$ , and the high- $q^2$  bins 14.18-16 GeV $^2$  and 16-19 GeV $^2$ . The SM theoretical expressions for all observables in  $B \rightarrow K^* \mu^+ \mu^-$  in the low and high- $q^2$  regions are given in [75], and are straightforwardly adapted to the VuQ model by modifying the values of the Wilson coefficients as in Eq. (2.29). The theoretical predictions for all the  $B \rightarrow K^* \mu^+ \mu^-$  observables are computed using the program **SuperIso** [49, 50]. For each bin, we compute the flavor observables and define the  $\chi^2$  as

$$\chi^2_{B \rightarrow K^* \mu^+ \mu^-} = \sum_{\text{bins}} \left[ \sum_{j \in (B \rightarrow K^* \mu^+ \mu^- \text{ obs.})} \left( \frac{O_j^{\text{exp}} - O_j^{\text{th}}}{\sigma_i} \right)^2 \right] \quad (2.38)$$

### 2.13 Branching ratio of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$

The quark-level transition  $b \rightarrow d\mu^+ \mu^-$  gives rise to the inclusive semi-leptonic decay  $B_d^0 \rightarrow X_d \mu^+ \mu^-$ , to exclusive semi-leptonic decays such as  $B_d^0 \rightarrow \pi^0 \mu^+ \mu^-$ , and also to the purely leptonic decay  $B_d^0 \rightarrow \mu^+ \mu^-$ . However, so far, none of these decays have been observed. We only have an upper bound on their branching ratios [78, 79]. Recently, LHCb has observed the  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$  decay with measured branching ratio of  $(2.3 \pm 0.6 \pm 0.1) \times 10^{-8}$  [24]. This is the first measurement of any decay channel induced by  $b \rightarrow d \mu^+ \mu^-$ .

The effective Hamiltonian for the process  $b \rightarrow d \mu^+ \mu^-$  and the modified Wilson coefficients in the VuQ model can be respectively obtained from Eqs. (2.28) and (2.29) by replacing  $s$  by  $d$ . The theoretical expression for  $\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$  is given in Ref. [80]. The contribution to  $\chi_{\text{total}}^2$  is

$$\chi_{B^+ \rightarrow \pi^+ \mu^+ \mu^-}^2 = \left( \frac{\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-) - 2.3 \times 10^{-8}}{0.66 \times 10^{-8}} \right)^2, \quad (2.39)$$

where, following Ref. [80], we have included a theoretical error of 10% in  $\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$ . This is due to uncertainties in the  $B^+ \rightarrow \pi^+$  form factors [81].

## 2.14 Branching ratio of $B_q \rightarrow \mu^+ \mu^-$ ( $q = s, d$ )

The branching ratio of  $B_q \rightarrow \mu^+ \mu^-$  in the VuQ model is given by

$$\mathcal{B}(B_q \rightarrow \mu^+ \mu^-) = \frac{G_F^2 \alpha^2 M_{B_q} m_\mu^2 f_{bq}^2 \tau_{B_q}}{16\pi^3} |V_{tq} V_{tb}^*|^2 \sqrt{1 - 4(m_\mu^2/M_{B_q}^2)} |C_{10}^{\text{tot},q}|^2, \quad (2.40)$$

where  $C_{10}^{\text{tot},s}$  is defined in Eq. (2.29), and  $C_{10}^{\text{tot},d}$  is given by

$$C_{10}^{\text{tot},d} = C_{10} + \frac{V_{t'd} V_{t'b}^*}{V_{td} V_{tb}^*} C_{10}''. \quad (2.41)$$

In order to include  $\mathcal{B}(B_q \rightarrow \mu^+ \mu^-)$  ( $q = s, d$ ) in the fit, we define

$$B_{\text{lepq}} = \frac{16\pi^3 \mathcal{B}(B_q \rightarrow \mu^+ \mu^-)}{G_F^2 \alpha^2 M_{B_q} m_\mu^2 f_{bq}^2 \tau_{B_q} \sqrt{1 - 4(m_\mu^2/M_{B_q}^2)}}. \quad (2.42)$$

Using the inputs given in Tables 1 and 3, we obtain

$$B_{\text{leps,exp}} = 0.025 \pm 0.006, \quad B_{\text{lepd,exp}} = 0.0048 \pm 0.0020. \quad (2.43)$$

The contribution to  $\chi_{\text{total}}^2$  from  $\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$  and  $\mathcal{B}(B_d^0 \rightarrow \mu^+ \mu^-)$  is then

$$\chi_{B_q \rightarrow \mu^+ \mu^-}^2 = \left( \frac{B_{\text{leps}} - 0.025}{0.006} \right)^2 + \left( \frac{B_{\text{lepd}} - 0.0048}{0.0020} \right)^2. \quad (2.44)$$

## 2.15 Branching ratio of $B \rightarrow \tau \bar{\nu}$

The branching ratio of  $B \rightarrow \tau \bar{\nu}$  is given by

$$\mathcal{B}(B \rightarrow \tau \bar{\nu}) = \frac{G_F^2 M_B m_\tau^2}{8\pi} \left( 1 - \frac{m_\tau^2}{M_B^2} \right)^2 f_{bd}^2 |V_{ub}|^2 \tau_{B^\pm}. \quad (2.45)$$

In order to include  $\mathcal{B}(B \rightarrow \tau \bar{\nu})$  in the fit, we define

$$B_{\text{Btau-nu}} = \frac{8\pi \mathcal{B}(B \rightarrow \tau \bar{\nu})}{G_F^2 M_B m_\tau^2 f_{bd}^2 \tau_B \sqrt{1 - m_\tau^2/M_B^2}}. \quad (2.46)$$

Using the inputs given in Tables 1 and 3, we obtain

$$B_{\text{Btau-nu,exp}} = (1.779 \pm 0.352) \times 10^{-5}. \quad (2.47)$$

The contribution to  $\chi_{\text{total}}^2$  from  $\mathcal{B}(B \rightarrow \tau \bar{\nu})$  is then

$$\chi_{B \rightarrow \tau \bar{\nu}}^2 = \left( \frac{B_{\text{Btau-nu}} - 1.779 \times 10^{-5}}{0.352 \times 10^{-5}} \right)^2. \quad (2.48)$$

## 2.16 Like-sign dimuon charge asymmetry $A_{SL}^b$

The (CP-violating) like-sign dimuon charge asymmetry in the  $B$  system is defined as

$$A_{SL}^b \equiv \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}}, \quad (2.49)$$

where  $N_b^{\pm\pm}$  is the number of events of  $b\bar{b} \rightarrow \mu^\pm \mu^\pm X$ . It can be written as

$$A_{SL}^b = c_{SL}^d A_{SL}^d + c_{SL}^s A_{SL}^s, \quad (2.50)$$

where  $A_{SL}^q = \text{Im}\left(\Gamma_{12}^{(q)}/M_{12}^{(q)}\right)$  ( $q = s, d$ ), with  $c_{SL}^d = 0.594 \pm 0.022$  and  $c_{SL}^s = 0.406 \pm 0.022$ . The theoretical expression for  $A_{SL}^q$  in the presence of NP is given in Ref. [82].

$A_{sl}^b$  has been measured by the DØ Collaboration. The measured value is  $(-4.96 \pm 1.53 \pm 0.72) \times 10^{-3}$  [29]. This deviates by  $2.7\sigma$  from the SM prediction of  $A_{SL}^b$  is  $(-2.44 \pm 0.42) \times 10^{-4}$ .

The quantities  $a$ ,  $b$  and  $c$  appear in the theoretical expressions for  $A_{SL}^q$  [82]. In computing the contribution to  $\chi^2$  from  $A_{SL}^b$ , one must include the errors in these quantities, as well as those in  $c_{SL}^d$  and  $c_{SL}^s$ . To do so, we consider all of these as parameters and add a contribution to  $\chi_{\text{total}}^2$ . To be precise,

$$\chi_{A_{SL}^b}^2 = \left( \frac{A_{SL}^b - (-4.96 \times 10^{-3})}{1.69 \times 10^{-3}} \right)^2 + \chi_c^2, \quad (2.51)$$

where

$$\begin{aligned} \chi_c^2 &= \left( \frac{c_{SL}^d - 0.594}{0.022} \right)^2 + \left( \frac{c_{SL}^s - 0.406}{0.022} \right)^2 \\ &\quad + \left( \frac{a - 10.5}{1.8} \right)^2 + \left( \frac{b - 0.2}{0.1} \right)^2 + \left( \frac{c - (-53.3)}{12} \right)^2. \end{aligned} \quad (2.52)$$

## 2.17 The oblique parameter $S$ and $T$

The theoretical expressions for the oblique parameters  $S$  and  $T$  in the VuQ model are given in Ref. [14]. For these non-decoupling corrections we define

$$\chi_{\text{Oblique}}^2 = \left( \frac{S - 0.0}{0.11} \right)^2 + \left( \frac{T - 0.02}{0.12} \right)^2. \quad (2.53)$$

## 3. Results of the fit

We first perform a  $\chi^2$  fit to obtain the Wolfenstein parameters of the standard CKM matrix. We then redo the fit, using the theoretical expressions of the VuQ model for the observables. We obtain values for the Wolfenstein parameters, as well as for the NP magnitudes  $P$ ,  $Q$  and  $r$  and the NP phases  $\delta_{t'd}$  and  $\delta_{t's}$ . The results of both fits are presented in Table 4, for  $m_{t'} = 800$  GeV and 1200 GeV.

Parameter	SM	$m_{t'} = 800 \text{ GeV}$	$m_{t'} = 1200 \text{ GeV}$
$\lambda$	$0.226 \pm 0.001$	$0.226 \pm 0.001$	$0.226 \pm 0.001$
$A$	$0.780 \pm 0.015$	$0.770 \pm 0.019$	$0.769 \pm 0.019$
$C$	$0.39 \pm 0.01$	$0.44 \pm 0.02$	$0.43 \pm 0.02$
$\delta_{ub}$	$1.21 \pm 0.08$	$1.13 \pm 0.11$	$1.15 \pm 0.09$
$P$	—	$0.40 \pm 0.26$	$0.30 \pm 0.21$
$Q$	—	$0.04 \pm 0.06$	$0.03 \pm 0.05$
$r$	—	$0.45 \pm 0.25$	$0.36 \pm 0.22$
$\delta_{t'd}$	—	$0.55 \pm 0.45$	$0.76 \pm 0.42$
$\delta_{t's}$	—	$0.52 \pm 3.26$	$0.96 \pm 1.21$
$\chi^2/d.o.f.$	$71.15/60$	$63.35/59$	$63.60/59$

Table 4: The results of the fits to the parameters of the CKM matrix in the SM and in the VuQ model.

Quantity	SM	$m_{t'} = 800 \text{ GeV}$	$m_{t'} = 1200 \text{ GeV}$
$ V_{ud} $	$0.9745 \pm 0.0002$	$0.9745 \pm 0.0002$	$0.9745 \pm 0.0002$
$ V_{us} $	$0.226 \pm 0.001$	$0.226 \pm 0.001$	$0.226 \pm 0.001$
$ V_{ub} $	$(3.52 \pm 0.13) \times 10^{-3}$	$(3.92 \pm 0.24) \times 10^{-3}$	$(3.85 \pm 0.21) \times 10^{-3}$
$ V_{cd} $	$0.226 \pm 0.001$	$0.226 \pm 0.001$	$0.226 \pm 0.001$
$ V_{cs} $	$0.9745 \pm 0.0002$	$0.9745 \pm 0.0002$	$0.9745 \pm 0.0002$
$ V_{cb} $	$0.040 \pm 0.001$	$0.039 \pm 0.001$	$0.039 \pm 0.001$
$ V_{td} $	$0.0084 \pm 0.0003$	$0.0078 \pm 0.0005$	$0.0080 \pm 0.0004$
$ V_{ts} $	$0.039 \pm 0.001$	$0.039 \pm 0.001$	$0.039 \pm 0.001$
$ V_{tb} $	1	$0.995 \pm 0.006$	$0.997 \pm 0.004$
$ V_{t'd} $	—	$0.005 \pm 0.003$	$0.003 \pm 0.002$
$ V_{t's} $	—	$0.002 \pm 0.003$	$0.001 \pm 0.002$
$ V_{t'b} $	—	$0.101 \pm 0.056$	$0.082 \pm 0.049$

Table 5: Magnitudes of the  $4 \times 3$  CKM matrix elements obtained from the fit.

From Table 4, it can be seen that the three-generation CKM parameters are not much affected by the addition of a vector isosinglet up-type quark  $t'$ . The allowed parameter space for  $C$  and  $\delta_{ub}$  expands a little as the constraints on  $|V_{ub}|$  coming from the unitarity of the  $3 \times 3$  CKM matrix are relaxed by the addition of the  $t'$  quark. The new real parameters,  $P$ ,  $Q$  and  $r$ , are consistent with zero. In addition, the vanishing of  $P$  and  $Q$  implies vanishing  $V_{t'd}$  and  $V_{t's}$ , respectively. In this case, the phases of these two elements have no significance.

The magnitudes of the elements of the  $4 \times 3$  CKM matrix, obtained using the fit values of Table 4, are given in Table 5. From this Table, we find that  $|V_{tb}| \geq 0.98$

Quantity	SM	$m_{t'} = 800$ GeV	$m_{t'} = 1200$ GeV
$ V_{td}V_{tb}^* $	$0.0084 \pm 0.0003$	$0.0077 \pm 0.0006$	$0.0079 \pm 0.0004$
$ V_{ts}V_{tb}^* $	$0.0391 \pm 0.0008$	$0.0387 \pm 0.0011$	$0.0386 \pm 0.001$
$ V_{td}V_{ts}^* $	$(0.33 \pm 0.02) \times 10^{-3}$	$(0.30 \pm 0.02) \times 10^{-3}$	$(0.30 \pm 0.02) \times 10^{-3}$
$ V_{t'd}V_{t'b}^* $	—	$(0.47 \pm 0.40) \times 10^{-3}$	$(0.28 \pm 0.26) \times 10^{-3}$
$ V_{t's}V_{t'b}^* $	—	$(0.19 \pm 0.32) \times 10^{-3}$	$(0.12 \pm 0.20) \times 10^{-3}$
$ V_{t'd}V_{t's}^* $	—	$(0.09 \pm 0.15) \times 10^{-4}$	$(0.05 \pm 0.09) \times 10^{-4}$

Table 6: In the VuQ model, combinations of CKM matrix elements that control mixing and decay in the  $B_d$ ,  $B_s$  and  $K$  sectors.

at  $3\sigma$ . Now, the direct measurement of  $|V_{tb}|$ , without assuming unitarity, has been performed using the single-top-quark production cross section. At the TeVatron one finds  $|V_{tb}| = 1.03 \pm 0.06$  [83, 84], while the LHC finds  $|V_{tb}| = 1.03 \pm 0.05$  [85, 86]. We therefore see that, although the present direct measurement of  $|V_{tb}|$  is consistent with the SM, a sizeable deviation from its SM value of 1 is not ruled out due to large experimental errors. On the other hand, we see that the constraints from present flavor-physics data do not allow such a sizeable deviation. We also find that the allowed values of all of the NP elements of the CKM matrix are consistent with zero. Furthermore, the  $3\sigma$  upper limits on these are  $|V_{t'd}| \leq 0.01$ ,  $|V_{t's}| \leq 0.01$  and  $|V_{t'b}| \leq 0.27$ , indicating that the mixing of  $t'$  quark with the other three quarks is constrained to be small.

The values of the magnitudes of the CKM factors that control mixing and decay in the  $B_d$ ,  $B_s$  and  $K$  sectors are given in Table 6. In the  $b \rightarrow s$  sector, the NP contribution is proportional to the CKM factor  $V_{t's}V_{t'b}^*$ . The corresponding CKM factor in the SM is  $V_{ts}V_{tb}^*$ . The fit indicates that  $|V_{t's}V_{t'b}^*| \ll |V_{ts}V_{tb}^*|$ . Thus, the NP contribution in the  $b \rightarrow s$  sector is tightly constrained in the VuQ model – large deviations from the SM predictions are not possible. This can be seen, for example, from the study of the  $B \rightarrow K^* \mu^+ \mu^-$  observable  $P'_5$  in the bin 4.3–8.68 GeV $^2$  (see Table 2). The disagreement between the experimental measurement of  $P'_5$  in this bin and its SM prediction is around the  $4\sigma$  level. In the SM fit, the  $\chi^2_{P'_5}$  contribution to the total  $\chi^2_{\min}$  is 16.73, reflecting the large discrepancy between measurement and prediction. In the VuQ fit, we find  $\chi^2_{P'_5} = 18.18$  for  $m_{t'} = 800$  GeV ( $\chi^2_{P'_5} = 17.36$  for  $m_{t'} = 1200$  GeV), which shows no improvement over the SM.

The situation is almost the same in the  $b \rightarrow d$  and  $s \rightarrow d$  sectors. It can be seen from Table 6 that both  $|V_{t'd}V_{t'b}^*|/|V_{td}V_{tb}^*|$  and  $|V_{t'd}V_{t's}^*|/|V_{td}V_{ts}^*|$  are of  $\mathcal{O}(10^{-1})$ . Thus the NP contributions in these sectors from the VuQ model are also expected to be small.

Observable	Predictions		
	SM	$m_{t'} = 800$ GeV	$m_{t'} = 1200$ GeV
$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \times 10^{11}$	$2.48 \pm 0.29$	$3.24 \pm 0.74$	$3.10 \pm 0.59$
$\mathcal{B}(B \rightarrow X_s \nu \bar{\nu}) \times 10^5$	$2.16 \pm 0.23$	$1.94 \pm 0.44$	$1.95 \pm 0.40$
$x_D$	Unknown	$\leq 0.08\%$ at $2\sigma$	$\leq 0.03\%$ at $2\sigma$
$\mathcal{B}(D \rightarrow \mu^+ \mu^-)$	$\approx 3 \times 10^{-13}$	$(4.56 \pm 10.01) \times 10^{-13}$	$(1.47 \pm 2.98) \times 10^{-13}$
$\mathcal{B}(t \rightarrow uZ)$	$\sim 10^{-17}$	$(1.34 \pm 2.19) \times 10^{-7}$	$(0.50 \pm 0.89) \times 10^{-7}$
$\mathcal{B}(t \rightarrow cZ)$	$\sim 10^{-14}$	$(1.03 \pm 2.69) \times 10^{-7}$	$(0.39 \pm 1.01) \times 10^{-7}$

Table 7: Predictions for observables in the VuQ model.

## 4. Predictions for other flavor-physics observables.

With the constraints found in the previous section for the NP CKM matrix elements, it is interesting to see whether any large deviations from the SM are possible in other flavor-physics observables. In this section, we provide predictions for some of the observables in the VuQ model. These are summarized in Table 7.

### 4.1 Branching fraction of $K_L \rightarrow \pi^0 \nu \bar{\nu}$

In the SM, the decay  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  is dominated by the short-distance loop diagrams with top-quark exchange, while the contributions due to the  $u$  and  $c$  quarks may be neglected. Thus, the  $t'$  quark in the loop may give a significant contribution. With the addition of the  $t'$ , the branching fraction of  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  can be written as [6, 7]

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \left( \frac{\text{Im}(V_{td} V_{ts}^*)}{\lambda^5} X(x_t) + \frac{\text{Im}(V_{t'd} V_{t's}^*)}{\lambda^5} X(x_{t'}) \right)^2, \quad (4.1)$$

with

$$\kappa_L = \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} \kappa_+ = (2.31 \pm 0.01) \times 10^{-10}. \quad (4.2)$$

The function  $X(x)$  ( $x \equiv m_{t,t'}^2/M_W^2$ ), relevant for the  $t$  and  $t'$  pieces, is given by

$$X(x) = \eta_X X_0(x), \quad (4.3)$$

where

$$X_0(x) = \frac{x}{8} \left[ -\frac{2+x}{1-x} + \frac{3x-6}{(1-x)^2} \ln x \right]. \quad (4.4)$$

Above,  $\eta_X$  is the NLO QCD correction; its value is estimated to be 0.994 [87].  $r_{K^+}$  summarizes the isospin-breaking corrections in relating  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  to  $K^+ \rightarrow \pi^0 e^+ \nu$ , while  $r_{K_L}$  summarizes the isospin breaking corrections in relating  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  to  $K^+ \rightarrow \pi^0 e^+ \nu$ .

$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  is a purely CP-violating quantity, i.e., it vanishes if CP is conserved. Thus, it is sensitive to non-standard CP-violating phases. Within the SM, the branching ratio of  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  can be predicted with very small uncertainties. It is given by [88, 89]

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (2.48 \pm 0.29) \times 10^{-11}. \quad (4.5)$$

The main source of uncertainty in the branching ratio prediction is the imaginary part of  $V_{td}$ . Other theoretical uncertainties are less than 2%. Experimentally, this decay has yet to be observed. The present upper bound on its branching ratio is  $2.6 \times 10^{-8}$  at 90% C.L. [90], which is about three orders of magnitude above its SM prediction. Given the constraints on the  $4 \times 3$  CKM matrix, the VuQ calculation predicts  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.24 \pm 0.74) \times 10^{-11}$  for  $m_{t'} = 800$  GeV ( $(3.10 \pm 0.59) \times 10^{-11}$  for  $m_{t'} = 1200$  GeV). At  $2\sigma$ ,  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 4.72 \times 10^{-11}$ , indicating that a large enhancement in the branching ratio is not allowed.

## 4.2 The branching fraction of $B \rightarrow X_s \nu \bar{\nu}$

In the SM, the decay  $B \rightarrow X_s \nu \bar{\nu}$  is dominated by the  $Z^0$  penguin and box diagrams involving top-quark exchange, and is theoretically clean. Therefore, we expect that any additional contributions due to a  $t'$  in the loop will be easily identifiable. The branching fraction for  $B \rightarrow X_s \nu \bar{\nu}$  in the presence of a  $t'$  quark is given by [6]

$$\mathcal{B}(B \rightarrow X_s \nu \bar{\nu}) = \frac{\alpha^2 \bar{\eta} \mathcal{B}(B \rightarrow X_c e \bar{\nu})}{2\pi^2 \sin^4 \theta_W |V_{cb}|^2 f(\hat{m}_c) \kappa(\hat{m}_c)} \left| V_{tb}^* V_{ts} X_0(x_t) \right|^2 \left| 1 + \frac{V_{t'b}^* V_{t's} X_0(x_{t'})}{V_{tb}^* V_{ts} X_0(x_t)} \right|^2. \quad (4.6)$$

The factor  $\bar{\eta} \approx 0.83$  represents the QCD correction to the matrix element of the  $b \rightarrow s \nu \bar{\nu}$  transition due to virtual and bremsstrahlung contributions,  $f(\hat{m}_c)$  is the phase-space factor in  $\mathcal{B}(B \rightarrow X_c e \bar{\nu})$ , and  $\kappa(\hat{m}_c)$  is the 1-loop QCD correction factor. The SM prediction for  $\mathcal{B}(B \rightarrow X_s \nu \bar{\nu})$  is  $(2.16 \pm 0.23) \times 10^{-5}$ , while in the VuQ model this value changes slightly to  $(1.94 \pm 0.44) \times 10^{-5}$  for  $m_{t'} = 800$  GeV ( $(1.95 \pm 0.40) \times 10^{-5}$  for  $m_{t'} = 1200$  GeV). Hence a large enhancement of the branching fraction of  $B \rightarrow X_s \nu \bar{\nu}$  is not allowed.

## 4.3 $D^0$ - $\bar{D}^0$ mixing

Within the SM,  $D^0$ - $\bar{D}^0$  mixing occurs at loop level and involves the lighter quarks  $d$ ,  $s$  and  $b$ . This implies a strong Glashow-Iliopoulos-Maiani (GIM) cancellation, and hence a small SD contribution. Furthermore, the  $b$ -quark contribution is highly suppressed,  $O(\lambda^8)$ , so that the mixing is dominated by the  $d$ - and  $s$ -quark contributions. There are, therefore, large LD contributions to  $D^0$ - $\bar{D}^0$  mixing, and indeed they dominate over the SD contributions. The present measurement of the  $D^0$ - $\bar{D}^0$  mixing parameter  $x_D$  is

$$x_D \equiv \frac{\Delta M_D}{\Gamma_D} = (0.8 \pm 0.1)\%. \quad (4.7)$$

This is much larger than the short-distance SM prediction. Still, in order to determine if the SM can explain this value of  $x_D$ , one must have an accurate estimate of the LD contribution. Unfortunately, this is not available at present.

As noted in the introduction, the mixing of the  $t'_L$  with  $\{u_L, c_L, t_L\}$  will induce tree-level  $Z$ -mediated FCNC's among the SM quarks. Thus, in the VuQ model,  $D^0$ - $\bar{D}^0$  mixing occurs at tree level. It may therefore provide a much larger contribution than that of the (short-distance) SM. Neglecting the SM contributions, in the VuQ model  $D^0$ - $\bar{D}^0$  mixing is given by [91]

$$x_d = \frac{G_F |U_{uc}|^2 f_D^2 M_D B_D r(m_c, M_Z)}{3\sqrt{2}\Gamma_D}, \quad (4.8)$$

where  $|U_{uc}| = V_{u4}V_{c4}^*$  is the  $Z$ - $u$ - $c$  flavor-changing coupling, and  $r(m_c, M_Z) = 0.778$  is the renormalization-group factor. Using  $f_D = 209.2 \pm 3.3$  MeV [41],  $B_D = 1.18 \pm 0.07$  [92] and  $\tau_D = 0.4101$  ps [31], we find that, given the constraints on  $V_{u4}V_{c4}^*$ , in the VuQ model,  $x_D = (0.016 \pm 0.034)\%$  for  $m_{t'} = 800$  GeV (( $0.005 \pm 0.010$ )% for  $m_{t'} = 1200$  GeV). Thus at  $2\sigma$ ,  $x_D \leq 0.08\%$ . We therefore see that the SD contribution in the VuQ model falls far below the observed value of  $D^0$ - $\bar{D}^0$  mixing.

#### 4.4 Branching fraction of $D^0 \rightarrow \mu^+ \mu^-$

Unlike  $D^0$ - $\bar{D}^0$  mixing, the SM prediction for the branching fraction of  $D^0 \rightarrow \mu^+ \mu^-$  can be estimated fairly accurately, even after including the LD contribution. The SM prediction for the  $D^0 \rightarrow \mu^+ \mu^-$  branching ratio is  $\approx 3 \times 10^{-13}$ , hence highly suppressed. Thus,  $D^0 \rightarrow \mu^+ \mu^-$  has the potential for large NP contributions. At present, we only have an experimental upper bound on the branching ratio:  $\mathcal{B}(D^0 \rightarrow \mu^+ \mu^-) \leq 7.6 \times 10^{-9}$  at 95% C.L. [93], which is several orders of magnitude larger than the SM prediction.

Within the VuQ model,  $D^0 \rightarrow \mu^+ \mu^-$  occurs at tree level due to  $Z$ -mediated FCNC's. Neglecting the SM contribution, the branching ratio in the VuQ model is given by [91]

$$\mathcal{B}(D^0 \rightarrow \mu^+ \mu^-) = \frac{G_F m_\mu^2 f_D^2 M_D}{32\pi\Gamma_D} \sqrt{1 - \frac{4m_\mu^2}{m_D^2}} |U_{uc}|^2. \quad (4.9)$$

For  $m_{t'} = 800$  GeV,  $\mathcal{B}(D^0 \rightarrow \mu^+ \mu^-) = (4.56 \pm 10.01) \times 10^{-13}$  (( $1.47 \pm 2.98$ )  $\times 10^{-13}$  for  $m_{t'} = 1200$  GeV). Thus, at  $2\sigma$ ,  $\mathcal{B}(D^0 \rightarrow \mu^+ \mu^-) \leq 2.46 \times 10^{-12}$ . We therefore observe that the branching ratio of  $D^0 \rightarrow \mu^+ \mu^-$  can be enhanced by an order of magnitude above its SM value, but this is still far below the present detection level.

#### 4.5 Branching fraction of $t \rightarrow qZ$ ( $q = c, u$ )

Within the SM, the branching ratios of the FCNC top decays  $t \rightarrow uZ$  and  $t \rightarrow cZ$  are  $\sim 10^{-17}$  and  $\sim 10^{-14}$ , respectively [94, 95]. The present upper bound on  $\mathcal{B}(t \rightarrow qZ)$

is 0.21% at 95% C.L. [96]. The discovery potential of  $\mathcal{B}(t \rightarrow qZ)$  is  $\sim 10^{-4}\text{-}10^{-5}$  at ATLAS and CMS. The SM value of  $\mathcal{B}(t \rightarrow qZ)$  is thus far below the detection level for these decays. This implies that these decays can only be observed if NP enhances their branching ratios by many orders of magnitude above their SM values.

This may be possible within the VuQ model, as here, due to  $Z$ -mediated FCNC's, these decays occur at tree level. Neglecting the SM contribution, the decay rate for  $t \rightarrow qZ$  is given by [95]

$$\Gamma(t \rightarrow qZ) = \frac{\alpha}{32 \sin^2 \theta_W \cos^2 \theta_W} |U_{qt}|^2 \frac{m_t^3}{M_Z^2} \left[ 1 - \frac{M_Z^2}{m_t^2} \right]^2 \left[ 1 + 2 \frac{M_Z^2}{m_t^2} \right], \quad (4.10)$$

where  $m_t = 173.2 \pm 0.9$  GeV [97] and  $|U_{qt}| = V_{q4} V_{t4}^*$ . As  $V_{tb}$  in this model is close to unity, we can approximate the top width by  $\Gamma(t \rightarrow bW^+)$ , which at leading order is given by

$$\Gamma(t \rightarrow bW^+) = \frac{\alpha}{16 \sin^2 \theta_W} |V_{tb}|^2 \frac{m_t^3}{M_W^2} \left[ 1 - 3 \frac{M_W^4}{m_t^4} + 2 \frac{M_W^6}{m_t^6} \right]. \quad (4.11)$$

The branching ratio of  $t \rightarrow qZ$  is therefore given by

$$\mathcal{B}(t \rightarrow qZ) = (0.463 \pm 0.001) \frac{|U_{qt}|^2}{|V_{tb}|^2}. \quad (4.12)$$

Using the values of parameters given in Table 4, we obtain  $|U_{ut}| = (0.53 \pm 0.43) \times 10^{-3}$  ( $(0.33 \pm 0.29) \times 10^{-3}$ ) and  $|U_{ct}| = (0.47 \pm 0.61) \times 10^{-3}$  ( $(0.29 \pm 0.37) \times 10^{-3}$ ) for  $m_{t'} = 800$  GeV (1200 GeV). This leads to  $\mathcal{B}(t \rightarrow uZ) = (1.34 \pm 2.19) \times 10^{-7}$  ( $(0.50 \pm 0.89) \times 10^{-7}$ ) and  $\mathcal{B}(t \rightarrow cZ) = (1.03 \pm 2.69) \times 10^{-7}$  ( $(0.39 \pm 1.01) \times 10^{-7}$ ) for  $m_{t'} = 800$  GeV (1200 GeV). Therefore, the FCNC branching ratios can indeed be enhanced by many orders of magnitude above their SM values. However, they are still two orders of magnitude below the present detection level for these decays.

## 5. Conclusions

In this paper we consider the VuQ model, in which a vector isosinglet up-type quark  $t'$  is added to the standard model (SM). In the VuQ model, the full CKM quark mixing matrix is  $4 \times 3$ , and is parametrized by four SM and five new-physics (NP) parameters. The NP parameters include three magnitudes and two (CP-violating) phases. We perform a fit using flavor-physics data to constrain all CKM parameters. The purpose is to determine whether there are any indications of NP, such as the non-unitarity of the  $3 \times 3$  SM CKM matrix, or, equivalently, nonzero values for some of the NP parameters. And even if there is no evidence of NP, we would like to ascertain whether sizeable NP effects are still possible in other flavor-physics observables, while being consistent with the constraints found in the fit.

The fit involves 68 flavor-physics observables. No evidence for NP is found: the values of the three NP magnitudes are consistent with zero, in which case the two NP phases have no significance. Specific results include the following:

- The deviations of the CKM matrix elements  $V_{ts}$  and  $V_{td}$  from their SM prediction are small.
- At  $3\sigma$ ,  $|V_{tb}| \geq 0.98$ . Any large deviation of  $|V_{tb}|$  from unity is therefore not possible in the VuQ model.
- The  $3\sigma$  upper limits on the new elements of the VuQ CKM matrix are:  $|V_{t'd}| \leq 0.01$ ,  $|V_{t's}| \leq 0.01$  and  $|V_{t'b}| \leq 0.27$ , indicating that the mixing of  $t'$  quark with the other three quarks is constrained to be small.

Turning to possible NP effects in the VuQ model, we find that any NP contributions to  $b \rightarrow s$ ,  $b \rightarrow d$  and  $s \rightarrow d$  transitions are tightly constrained. We also find,

- A large enhancement of SD contribution to  $x_d$  (i.e.,  $D^0$ - $\bar{D}^0$  mixing) is not allowed.
- The branching ratio of  $D^0 \rightarrow \mu^+ \mu^-$  can be enhanced by an order of magnitude above its SM value, but this is still far below the present detection level.
- The branching ratios of the flavor-changing decays  $t \rightarrow qZ$  ( $q = c, u$ ) can be enhanced by many orders of magnitude. However, they are still two orders of magnitude below the present detection level.

In summary, current flavor data puts extremely stringent constraints on the VuQ model. There are no hints of NP in the CKM matrix. Furthermore, the fit to the data indicates that any VuQ contributions to loop-level flavor-changing  $b \rightarrow s$ ,  $b \rightarrow d$  and  $s \rightarrow d$  transitions are very small. There can be significant enhancements of the branching ratios of  $t \rightarrow uZ$  and  $t \rightarrow cZ$  decays, but these are still below detection levels.

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